Review of *Circulant Matrices* by Philip Davis*

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This book brings a fresh new slant to an old topic. Circulant matrices are determined by the elements of the first row, which are shifted in each succeeding row one place to the right. They have been studied for well over a century, and one would think perhaps that everything worth saying about them had already been said, but one would be wrong. Most of the best-known results on circulants have been included in this book, but the author brings imagination and creative insight to the subject, and thus adds a new dimension. What to the standard linear algebraist is a vector with complex elements becomes in this book an "n-gon," a polygon in the complex plane whose vertices are the elements of the n-tuple. Intriguing things happen to this n-gon under the transformations defined by multiplication by certain types of circulants.

The name of the book is perhaps misleading, for it is far more than just an essay on properties of circulants. An astonishing amount of matrix theory, both old and new, is contained in this small volume. Much of the material in the first four chapters could be used to enrich an intermediate course in linear algebra at the undergraduate level. In addition to the geometry and algebra of circulants other topics are included, such as block matrices, Kronecker products, the UDV theorem and generalized inverses. These are not usually found in standard books on linear algebra and matrices.

More difficult material is introduced in the last two chapters, and these would be an excellent supplement for a graduate seminar. Chapter 5 contains a number of generalizations of circulants which have been the topics of research papers in the past two decades. These include block circulants; block circulants with circulant blocks, which are in turn generalized to level q circulants; retrocirculants; g-circulants; k-circulants; skew circulants; and others.

In the final chapter the author gives what he calls the "leitmotiv" of the book. This plays two roles, first to unify the results on all the different types of circulants discussed previously, and secondly to present a generalization of

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circulant theory along the lines of centralizers. The last section builds up to least squares approximations of matrices by certain special matrices which are elements of centralizers. Throughout the book there are excellent and challenging problems of various degrees of difficulty.

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